

APM 346 Calculus of Variations Problems.

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The following problems are taken directly from [Str08, Ch.14.3]. Number in the problem are the problem numbers from Strauss.

Problem 1 (14.3.1). *Use calculus of variations to prove that the shortest path between two points is a straight line.*

Problem 2 (14.3.4). *Find the curve $y = u(x)$ that makes the integral $\int_0^1 (u'^2 + xu) dx$ stationary subject to the constraints $u(0) = 0$ and $u(1) = 1$.*

Problem 3 (14.3.10). (a) *If the action is $A[u] = \iint (\frac{1}{2}u_x u_t + u_x^3 - \frac{1}{2}u_{xx}^2) dx dt$, find the Euler-Lagrange equation.*

(b) *If $v = u_x$, show that v satisfies the KdV equation.¹*

In order to solve the following problem, read the section on Lagrange multipliers in [IvrXX]:

Problem 4 (14.3.2). *Find the shortest curve in the xy -plane that joins the two given points $(0, a)$ and $(0, b)$ and that has given area A below it (above the x -axis and between $x = 0$ and $x = 1$); a and b are positive.*

References

[IvrXX] Victor Ivrii. Partial Differential Equations. online textbook for APM346, 20XX.

[Str08] Walter A. Strauss. *Partial differential equations*. John Wiley & Sons, Ltd., Chichester, second edition, 2008. An introduction.

¹Remember the KdV equation? It was Example 9 in Lecture 1!